



Matrix Multiplication

R Horan & M Lavelle

The aim of this document is to provide a short, self assessment programme for students who wish to learn how to multiply matrices.

Table of Contents

1. Introduction
2. Matrix Multiplication 1
3. Matrix Multiplication 2
4. The Identity Matrix
5. Quiz on Matrix Multiplication
Solutions to Exercises
Solutions to Quizzes

The full range of these packages and some instructions, should they be required, can be obtained from our web page [Mathematics Support Materials](#).

1. Introduction

In the package **Introduction to Matrices** the basic rules of *addition* and *subtraction* of matrices, as well as *scalar multiplication*, were introduced. The rule for the *multiplication of two matrices* is the subject of this package. The first example is the simplest.

Recall that if M is a matrix then the transpose of M , written M^T , is the matrix obtained from M by writing the rows of M as the columns of M^T .

If $A = (a_1 \ a_2 \ \dots \ a_n)$ is a $1 \times n$ (row) matrix and $B = (b_1 \ b_2 \ \dots \ b_n)^T$ is a $n \times 1$ (column) matrix then the product AB is defined as

$$AB = (a_1 \ a_2 \ \dots \ a_n) \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} = a_1 b_1 + a_2 b_2 + \dots + a_n b_n$$

This general rule is sometimes called the *inner product*.

N.B. The *row matrix* is on the left and the *column matrix* is on the right.

Example 1 In each of the following cases, find the product AB .

(a) $A = (1 \ 2)$, $B = (4 \ 3)^T$. (b) $A = (1 \ 1 \ 1)$, $B = (2 \ 3 \ 4)^T$.

(c) $A = (1 \ -1 \ 2 \ 3)$, $B = (1 \ 1 \ -3 \ 2)^T$.

Solution

(a) $AB = (1 \ 2) \begin{pmatrix} 4 \\ 3 \end{pmatrix} = 1 \times 4 + 2 \times 3 = 4 + 6 = 10.$

(b) $AB = (1 \ 1 \ 1) \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} = 1 \times 2 + 1 \times 3 + 1 \times 4 = 2 + 3 + 4 = 9.$

(c) $AB = (1 \ -1 \ 2 \ 3) \begin{pmatrix} 1 \\ 1 \\ -3 \\ 2 \end{pmatrix} = 1 \times 1 + 1 \times (-1) + 2 \times (-3) + 3 \times 2$
 $= 1 + (-1) + (-6) + 6 = 0.$

EXERCISE 1. For each of the cases below, calculate AB . (Click on the green letters for solutions.)

(a) $A = (-2 \ 4)$, $B = (3 \ 2)^T$,

(b) $A = (5 \ 3 \ -2)$, $B = (3 \ -4 \ 2)^T$,

(c) $A = (4 \ 4 \ -2 \ -3)$, $B = (5 \ -4 \ 3 \ 2)^T$.

The following observations are worth noting.

- The row matrix is on the left, the column matrix is on the right.
- The row and column have the same number of elements.
- The inner product AB is a 1×1 matrix, i.e. a *number*.
- Nothing has yet been said about a matrix product BA .

Quiz If $A = (x \ x \ 1)$ and $B = (x \ 6 \ 9)^T$, which of the following values of x will result in $AB = 0$?

(a) $x = 1$, (b) $x = 3$, (c) $x = -3$, (d) $x = -2$.

2. Matrix Multiplication 1

The previous section gave the rule for the multiplication of a row vector A with a column vector B , the *inner product* AB . This section will extend this idea to more general matrices.

Suppose that $A = \begin{pmatrix} a_1 & a_2 & \dots & a_n \\ c_1 & c_2 & \dots & c_n \end{pmatrix}$ and $B = (b_1 \ b_2 \ \dots \ b_n)^T$.

Then

$$AB = \begin{pmatrix} a_1 & a_2 & \dots & a_n \\ c_1 & c_2 & \dots & c_n \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} = \begin{pmatrix} a_1 b_1 + a_2 b_2 + \dots + a_n b_n \\ c_1 b_1 + c_2 b_2 + \dots + c_n b_n \end{pmatrix}$$

Example 2 Find AB for each of the following cases.

(a) $A = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$, $B = (4 \ 3)^T$.

(b) $A = \begin{pmatrix} 1 & 1 & 1 \\ -2 & 1 & -3 \end{pmatrix}$, $B = (2 \ 3 \ 4)^T$.

Solution

$$(a) AB = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \times 4 + 2 \times 3 \\ 3 \times 4 + (-1) \times 3 \end{pmatrix} = \begin{pmatrix} 10 \\ 9 \end{pmatrix}$$

$$(b) AB = \begin{pmatrix} 1 & 1 & 1 \\ -2 & 1 & -3 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \times 2 + 1 \times 3 + 1 \times 4 \\ (-2) \times 2 + 1 \times 3 + (-3) \times 4 \end{pmatrix} = \begin{pmatrix} 9 \\ -13 \end{pmatrix}$$

The following observations on AB are worth noting.

- The element in the *first row* of AB is the *inner product* of the *first row* of A with the column matrix B .
- The element in the *second row* of AB is the *inner product* of the *second row* of A with the column matrix B .
- The number of *columns* of A must be equal to the number of *rows* of B .
- If A is $2 \times n$ and B is $n \times 1$ then AB is 2×1 .

This rule for multiplication may be extended to matrices, A , which have more than two rows. For example, if A had 3 rows then the resulting matrix, AB , would have a third row; the value of this element would be the *inner product* of the *third row* of A with the column matrix B .

EXERCISE 2. For each of the cases below, calculate AB . (Click on the green letters for solutions.)

$$(a) \quad A = \begin{pmatrix} -2 & 4 \\ 5 & 3 \end{pmatrix}, \quad B = (4 \ 3)^T.$$

$$(b) \quad A = \begin{pmatrix} 5 & 3 & 2 \\ 4 & -1 & -1 \end{pmatrix}, \quad B = (2 \ 3 \ 4)^T.$$

$$(c) \quad A = \begin{pmatrix} -2 & 4 \\ 5 & 3 \\ 4 & -1 \end{pmatrix}, \quad B = (4 \ 3)^T.$$

$$(d) \quad A = \begin{pmatrix} 4 & 4 & -2 & -3 \\ 3 & -1 & -1 & 2 \end{pmatrix}, \quad B = (5 \ -4 \ 3 \ 2)^T.$$

3. Matrix Multiplication 2

The extension of the concept of matrix multiplication to matrices, A , B , in which A has more than one row and B has more than one column is now possible. The product matrix AB will have the same number of columns as B and each column is obtained by taking the product of A with each column of B , in turn, as shown below.

Let $A = \begin{pmatrix} 4 & 1 \\ 2 & 3 \\ -1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$ and let b_1 , b_2 be the first and

second columns of B respectively. Then

$$Ab_1 = \begin{pmatrix} 4 & 1 \\ 2 & 3 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 11 \\ 13 \\ 4 \end{pmatrix} \quad \text{and} \quad Ab_2 = \begin{pmatrix} 4 & 1 \\ 2 & 3 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \\ 5 \end{pmatrix}.$$

Thus

$$AB = \begin{pmatrix} 4 & 1 \\ 2 & 3 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 11 & -2 \\ 13 & 4 \\ 4 & 5 \end{pmatrix}.$$

EXERCISE 3. For each of the cases below, calculate AB . (Click on the green letters for solutions.)

$$(a) \quad A = \begin{pmatrix} -2 & 4 \\ 5 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} -2 & 4 \\ 5 & 3 \end{pmatrix}.$$

$$(b) \quad A = \begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix}, \quad B = \begin{pmatrix} 5 & -2 \\ -7 & 3 \end{pmatrix}.$$

$$(c) \quad A = \begin{pmatrix} -2 & 4 \\ 5 & 3 \\ 4 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} -2 & 4 \\ 5 & 3 \end{pmatrix}.$$

$$(d) \quad A = \begin{pmatrix} 5 & 3 & 2 \\ 4 & -1 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} -2 & 4 \\ 5 & 3 \\ 4 & -1 \end{pmatrix}.$$

NB The rules for finding the product of two matrices are summarised on the next page.

- If A is $m \times n$ and B is $n \times r$ then the product AB exists.
- The resulting matrix is $m \times r$. $((m \times n)(n \times r) = m \times r)$
- The element in the i th row, j th column of the matrix AB is the *inner product* of the i th row of A with the j th column of B .

Example 3 Find the element in the *2nd* row *3rd* column of AB if

$$A = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 1 & 4 & -2 \\ 3 & -1 & 2 \end{pmatrix}.$$

Solution Since A is 2×2 and B is 2×3 , the product AB exists and is a 2×3 matrix. The required element is the inner product of the *second row* of A with the *third column* of B , i.e.

$$(-1) \times (-2) + 3 \times 2 = 2 + 6 = 8.$$

EXERCISE 4. If

$$A = \begin{pmatrix} 1 & -2 & 4 & 5 \\ 7 & -8 & -6 & 2 \\ 2 & 3 & -1 & -2 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -2 & 1 & -3 \\ 0 & -2 & -5 \\ 4 & -3 & -7 \\ 0 & 0 & 6 \end{pmatrix}$$

find the ij element, *i.e.* the element in the i th row j th column, of AB for the following cases. (Click on the green letters for solutions.)

- (a) $i = 3, j = 2$, (b) $i = 2, j = 3$, (c) $i = 1, j = 2$,
(d) $i = 2, j = 1$, (e) $i = 3, j = 1$, (f) $i = 1, j = 3$,

Quiz Which of the following is the element in the 3rd row, 3rd column, of the matrix AB in the above exercise?

- (a) 26, (b) -26, (c) -12, (d) 12.

4. The Identity Matrix

If A and B are two matrices, the product AB can be found if the number of *columns* of A equals the number of *rows* of B . If A is 2×3 and B is 3×5 then AB can be calculated but BA does not exist. The *order* in which matrices are multiplied together matters. Even when AB and BA both exist it is usually the case that $AB \neq BA$.

There is one particular matrix, the *identity matrix*, which has very special multiplication properties. The $n \times n$ *identity matrix* is the $n \times n$ matrix with 1s and 0s *as shown below*.

Example 4 The 2×2 , 3×3 and 4×4 identity matrices are

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

The most important property of the identity matrix is revealed in the following exercise.

EXERCISE 5. If the *identity matrix* is denoted by I and the matrix M is

$$M = \begin{pmatrix} 1 & 2 & 4 \\ 7 & 8 & 6 \end{pmatrix},$$

use the appropriate identity matrix to calculate the following matrix products. (Click on the green letters for solutions.)

- (a) IM , where I is the 2×2 identity matrix, (b) MI , where I is the 3×3 identity matrix.

In matrix multiplication the identity matrix, I , behaves exactly like the number 1 in ordinary multiplication. This was seen in the previous exercise. For part (a), the matrix I is the 2×2 identity matrix; in part (b), I was 3×3 ; they satisfy the equation $IM = M = MI$.

Example 5 The matrices A , B are

$$A = \begin{pmatrix} 4 & 3 & 2 \\ 5 & 6 & 3 \\ 3 & 5 & 2 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 3 & -4 & 3 \\ 1 & -2 & 2 \\ -7 & 11 & -9 \end{pmatrix}.$$

Calculate AB and BA .

Solution Using the rules of matrix multiplication,

$$AB = \begin{pmatrix} 4 & 3 & 2 \\ 5 & 6 & 3 \\ 3 & 5 & 2 \end{pmatrix} \begin{pmatrix} 3 & -4 & 3 \\ 1 & -2 & 2 \\ -7 & 11 & -9 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I.$$

$$BA = \begin{pmatrix} 3 & -4 & 3 \\ 1 & -2 & 2 \\ -7 & 11 & -9 \end{pmatrix} \begin{pmatrix} 4 & 3 & 2 \\ 5 & 6 & 3 \\ 3 & 5 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I.$$

The matrix B is the *inverse* of the matrix A , and this is usually written as A^{-1} . Equally, the matrix A is the *inverse* of the matrix B . The equation $AA^{-1} = A^{-1}A = I$ is always true.

5. Quiz on Matrix Multiplication

Let $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 1 & 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ -2 & 1 & 2 \end{pmatrix}$, $C = \begin{pmatrix} 3 & 1 & 4 \\ 2 & 1 & 3 \\ 2 & 2 & 1 \end{pmatrix}$.

Choose the correct option from the following.

Begin Quiz

- The 2×3 element of AB is
(a) -1 , (b) 1 , (c) 0 , (d) 2 .
- The 3×1 element of CB is
(a) 3 , (b) -1 , (c) 4 , (d) -6 .
- The 2×2 element of CA is
(a) 4 , (b) -3 , (c) 0 , (d) 2 .
- (a) $B = C^{-1}$, (b) $A = B^{-1}$, (c) $C = A^{-1}$

End Quiz

Solutions to Exercises

Exercise 1(a)

If the row matrix $A = (-2 \ 4)$ and the column matrix

$$B = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

are multiplied, the resulting **inner product** is

$$\begin{aligned} AB &= (-2 \ 4) \begin{pmatrix} 3 \\ 2 \end{pmatrix} &= & -2 \times 3 + 4 \times 2 \\ & &= & -6 + 8 \\ & &= & 2. \end{aligned}$$

Click on the green square to return



Exercise 1(b)

If the row matrix $A = (5 \ 3 \ -2)$ and the column matrix

$$B = \begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix}$$

are multiplied, the resulting **inner product** is

$$\begin{aligned} AB &= (5 \ 3 \ -2) \begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix} = 5 \times 3 + 3 \times (-4) + (-2) \times 2 \\ &= 15 - 12 - 4 = -1. \end{aligned}$$

Click on the green square to return



Exercise 1(c)

If the row matrix $A = (4 \ 4 \ -2 \ -3)$ and the column matrix

$$B = \begin{pmatrix} 5 \\ -4 \\ 3 \\ -2 \end{pmatrix}$$

are multiplied, their inner product AB is

$$\begin{aligned} (4 \ 4 \ -2 \ -3) \begin{pmatrix} 5 \\ -4 \\ 3 \\ -2 \end{pmatrix} &= 4 \times 5 + 4 \times (-4) + (-2) \times 3 + (-3) \times (-2) \\ &= 20 - 16 - 6 + 6 = 4. \end{aligned}$$

Click on the green square to return



Exercise 2(a)

For the 2×2 matrix

$$A = \begin{pmatrix} -2 & 4 \\ 5 & 3 \end{pmatrix}$$

and the column $B = (4 \ 3)^T$, the product AB is

$$AB = \begin{pmatrix} -2 & 4 \\ 5 & 3 \end{pmatrix} \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} (-2) \times 4 + 4 \times 3 \\ 5 \times 4 + 3 \times 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 29 \end{pmatrix}.$$

Click on the green square to return



Exercise 2(b)

If the 2×3 matrix

$$A = \begin{pmatrix} 5 & 3 & 2 \\ 4 & -1 & -1 \end{pmatrix}$$

and the column matrix $B = (2 \ 3 \ 4)^T$ are multiplied together, then the resulting product AB is

$$\begin{aligned} AB &= \begin{pmatrix} 5 & 3 & 2 \\ 4 & -1 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 5 \times 2 + 3 \times 3 + 2 \times 4 \\ 4 \times 2 + (-1) \times 3 + (-1) \times 4 \end{pmatrix} \\ &= \begin{pmatrix} 27 \\ 1 \end{pmatrix}. \end{aligned}$$

Click on the green square to return



Exercise 2(c)

If the 3×2 matrix is

$$A = \begin{pmatrix} -2 & 4 \\ 5 & 3 \\ 4 & -1 \end{pmatrix}$$

and the column matrix is $B = (4 \ 3)^T$, then the product AB is

$$AB = \begin{pmatrix} -2 & 4 \\ 5 & 3 \\ 4 & -1 \end{pmatrix} \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} (-2) \times 4 + 4 \times 3 \\ 5 \times 4 + 3 \times 3 \\ 4 \times 4 + (-1) \times 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 29 \\ 13 \end{pmatrix}$$

Click on the green square to return



Exercise 2(d)

If the 2×4 matrix

$$A = \begin{pmatrix} 4 & 4 & -2 & -3 \\ 3 & -1 & -1 & 2 \end{pmatrix}$$

is multiplied with the column matrix $B = (5 \ -4 \ 3 \ 2)^T$, the resulting product, AB , is

$$AB = \begin{pmatrix} 4 & 4 & -2 & -3 \\ 3 & -1 & -1 & 2 \end{pmatrix} \begin{pmatrix} 5 \\ -4 \\ 3 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \times 5 + 4 \times (-4) + (-2) \times 3 + (-3) \times 2 \\ 3 \times 5 + (-1) \times (-4) + (-1) \times 3 + 2 \times 2 \end{pmatrix} = \begin{pmatrix} -8 \\ 20 \end{pmatrix}.$$

Click on the green square to return



Exercise 3(a)

Let A and B be the 2×2 matrices:

$$A = \begin{pmatrix} -2 & 4 \\ 5 & 3 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -2 & 4 \\ 5 & 3 \end{pmatrix}.$$

The matrix AB is

$$\begin{aligned} AB &= \begin{pmatrix} -2 & 4 \\ 5 & 3 \end{pmatrix} \begin{pmatrix} -2 & 4 \\ 5 & 3 \end{pmatrix} = \begin{pmatrix} -2 \times (-2) + 4 \times 5 & -2 \times 4 + 4 \times 3 \\ 5 \times (-2) + 3 \times 5 & 5 \times 4 + 3 \times 3 \end{pmatrix} \\ &= \begin{pmatrix} 24 & 4 \\ 5 & 29 \end{pmatrix}. \end{aligned}$$

Click on the green square to return



Exercise 3(b)

If A and B are the 2×2 matrices:

$$A = \begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 5 & -2 \\ -7 & 3 \end{pmatrix},$$

then the matrix product AB is

$$\begin{aligned} AB &= \begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix} \begin{pmatrix} 5 & -2 \\ -7 & 3 \end{pmatrix} = \begin{pmatrix} 3 \times 5 + 2 \times (-7) & 3 \times (-2) + 2 \times 3 \\ 7 \times 5 + 5 \times (-7) & 7 \times (-2) + 5 \times 3 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \end{aligned}$$

This is called the 2×2 **identity matrix**.

Click on the green square to return



Exercise 3(c)

If A and B are the matrices

$$A = \begin{pmatrix} -2 & 4 \\ 5 & 3 \\ 4 & -1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -2 & 4 \\ 5 & 3 \end{pmatrix}$$

then the matrix product AB is

$$\begin{aligned} AB &= \begin{pmatrix} -2 & 4 \\ 5 & 3 \\ 4 & -1 \end{pmatrix} \begin{pmatrix} -2 & 4 \\ 5 & 3 \end{pmatrix} = \begin{pmatrix} (-2) \times (-2) + 4 \times 5 & (-2) \times 4 + 4 \times 3 \\ 5 \times (-2) + 3 \times 5 & 5 \times 4 + 3 \times 3 \\ 4 \times (-2) + (-1) \times 5 & 4 \times 4 + (-1) \times 3 \end{pmatrix} \\ &= \begin{pmatrix} 24 & 4 \\ 5 & 29 \\ -13 & 13 \end{pmatrix}. \end{aligned}$$

Click on the green square to return



Exercise 3(d)

If A is 2×3 and B be is 3×2 given by the following

$$A = \begin{pmatrix} 5 & 3 & 2 \\ 4 & -1 & -1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -2 & 4 \\ 5 & 3 \\ 4 & -1 \end{pmatrix},$$

then the matrix product AB is

$$\begin{aligned} AB &= \begin{pmatrix} 5 & 3 & 2 \\ 4 & -1 & -1 \end{pmatrix} \begin{pmatrix} -2 & 4 \\ 5 & 3 \\ 4 & -1 \end{pmatrix}, \\ &= \begin{pmatrix} 5 \times (-2) + 3 \times 5 + 2 \times 4 & 5 \times 4 + 3 \times 3 + 2 \times (-1) \\ 4 \times (-2) + (-1) \times 5 + (-1) \times 4 & 4 \times 4 + (-1) \times 3 + (-1) \times (-1) \end{pmatrix} \\ &= \begin{pmatrix} 13 & 27 \\ -17 & 14 \end{pmatrix}. \end{aligned}$$

Click on the green square to return



Exercise 4(a)

If $A = \begin{pmatrix} 1 & -2 & 4 & 5 \\ 7 & -8 & -6 & 2 \\ 2 & 3 & -1 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 1 & -3 \\ 0 & -2 & -5 \\ 4 & -3 & -7 \\ 0 & 0 & 6 \end{pmatrix}$,

the (32) element in the matrix AB , $(AB)_{32}$, is the inner product of the *third row* of A with the *second column* of B , i.e.

$$\begin{aligned}(AB)_{32} &= 2 \times 1 + 3 \times (-2) + (-1) \times (-3) + (-2) \times 0 \\ &= 2 - 6 + 3 + 0 = -1.\end{aligned}$$

Click on the green square to return



Exercise 4(b)

If $A = \begin{pmatrix} 1 & -2 & 4 & 5 \\ 7 & -8 & -6 & 2 \\ 2 & 3 & -1 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 1 & -3 \\ 0 & -2 & -5 \\ 4 & -3 & -7 \\ 0 & 0 & 6 \end{pmatrix}$,

the (23) element of AB , $(AB)_{23}$, is the inner product of the *second row* of A with the *third column* of B , i.e.

$$\begin{aligned}(AB)_{23} &= 7 \times (-3) + (-8) \times (-5) + (-6) \times (-7) + 2 \times 6 \\ &= -21 + 40 + 42 + 12 = 73.\end{aligned}$$

Click on the green square to return



Exercise 4(c)

$$\text{If } A = \begin{pmatrix} 1 & -2 & 4 & 5 \\ 7 & -8 & -6 & 2 \\ 2 & 3 & -1 & -2 \end{pmatrix} \text{ and } B = \begin{pmatrix} -2 & 1 & -3 \\ 0 & -2 & -5 \\ 4 & -3 & -7 \\ 0 & 0 & 6 \end{pmatrix},$$

the (12) element in the matrix AB , $(AB)_{12}$, is the inner product of the *first row* of A with the *second column* of B , i.e.

$$\begin{aligned} (AB)_{12} &= 1 \times 1 + (-2) \times (-2) + 4 \times (-3) + 5 \times 0 \\ &= 1 + 4 - 12 + 0 \\ &= -7. \end{aligned}$$

Click on the green square to return



Exercise 4(d)

$$\text{If } A = \begin{pmatrix} 1 & -2 & 4 & 5 \\ 7 & -8 & -6 & 2 \\ 2 & 3 & -1 & -2 \end{pmatrix} \text{ and } B = \begin{pmatrix} -2 & 1 & -3 \\ 0 & -2 & -5 \\ 4 & -3 & -7 \\ 0 & 0 & 6 \end{pmatrix},$$

the (21) element in the matrix AB , $(AB)_{21}$, is the inner product of the *second row* of A with the *first column* of B , i.e.

$$\begin{aligned} (AB)_{21} &= 7 \times (-2) + (-8) \times 0 + (-6) \times 4 + 2 \times 0 \\ &= -14 + 0 - 24 + 0 = -38. \end{aligned}$$

Click on the green square to return



Exercise 4(e)

$$\text{If } A = \begin{pmatrix} 1 & -2 & 4 & 5 \\ 7 & -8 & -6 & 2 \\ 2 & 3 & -1 & -2 \end{pmatrix} \text{ and } B = \begin{pmatrix} -2 & 1 & -3 \\ 0 & -2 & -5 \\ 4 & -3 & -7 \\ 0 & 0 & 6 \end{pmatrix},$$

the (31) element in the matrix AB , $(AB)_{31}$, is the inner product of the *third row* of A with the *first column* of B , i.e.

$$\begin{aligned} (AB)_{31} &= 2 \times (-2) + 3 \times 0 + (-1) \times 4 + (-2) \times 0 \\ &= -4 + 0 - 4 + 0 = -8. \end{aligned}$$

Click on the green square to return



Exercise 4(f)

$$\text{If } A = \begin{pmatrix} 1 & -2 & 4 & 5 \\ 7 & -8 & -6 & 2 \\ 2 & 3 & -1 & -2 \end{pmatrix} \text{ and } B = \begin{pmatrix} -2 & 1 & -3 \\ 0 & -2 & -5 \\ 4 & -3 & -7 \\ 0 & 0 & 6 \end{pmatrix},$$

the (13) element in the matrix AB , $(AB)_{13}$, is the inner product of the *first row* of A with the *third column* of B , i.e.

$$\begin{aligned} (AB)_{13} &= 1 \times (-3) + (-2) \times (-5) + 4 \times (-7) + 5 \times 6 \\ &= -3 + 10 - 28 + 30 = 9. \end{aligned}$$

Click on the green square to return



Exercise 5(a)

For the 2×3 matrix $M = \begin{pmatrix} 1 & 2 & 4 \\ 7 & 8 & 6 \end{pmatrix}$,

the **left identity** matrix (multiplying M on the **left** to obtain IM) is the 2×2 matrix I :

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

The product IM is then

$$\begin{aligned} IM &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 4 \\ 7 & 8 & 6 \end{pmatrix} \\ &= \begin{pmatrix} 1 \times 1 + 0 \times 7 & 1 \times 2 + 0 \times 8 & 1 \times 4 + 0 \times 6 \\ 0 \times 1 + 1 \times 7 & 0 \times 2 + 1 \times 8 & 0 \times 4 + 1 \times 6 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 2 & 4 \\ 7 & 8 & 6 \end{pmatrix} = M. \end{aligned}$$

Click on the green square to return



Exercise 5(b)

For the 2×3 matrix $M = \begin{pmatrix} 1 & 2 & 4 \\ 7 & 8 & 6 \end{pmatrix}$, the **right identity** matrix (multiplying M on the **right** to obtain MI) is the 3×3 matrix I :

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

The product MI is thus

$$\begin{aligned} MI &= \begin{pmatrix} 1 & 2 & 4 \\ 7 & 8 & 6 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 \times 1 + 2 \times 0 + 4 \times 0 & 1 \times 0 + 2 \times 1 + 4 \times 0 & 1 \times 0 + 2 \times 0 + 4 \times 1 \\ 7 \times 1 + 8 \times 0 + 6 \times 0 & 7 \times 0 + 8 \times 1 + 6 \times 0 & 7 \times 0 + 8 \times 0 + 6 \times 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 2 & 4 \\ 7 & 8 & 6 \end{pmatrix} = M. \end{aligned}$$

Click on the green square to return



Solutions to Quizzes

Solution to Quiz:

Multiplying the row matrix $A = (x \ x \ 1)$ with the column matrix

$$B = \begin{pmatrix} x \\ 6 \\ 9 \end{pmatrix}$$

from the left we have

$$\begin{aligned} AB &= (x \ x \ 1) \begin{pmatrix} x \\ 6 \\ 9 \end{pmatrix} = x \times x + x \times 6 + 1 \times 9 \\ &= x^2 + 6x + 9 = (x + 3)^2. \end{aligned}$$

Therefore the inner product $AB = 0$, if $x = -3$.

End Quiz

Solution to Quiz:

The matrices A and B from Exercise 4 are

$$A = \begin{pmatrix} 1 & -2 & 4 & 5 \\ 7 & -8 & -6 & 2 \\ 2 & 3 & -1 & -2 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -2 & 1 & -3 \\ 0 & -2 & -5 \\ 4 & -3 & -7 \\ 0 & 0 & 6 \end{pmatrix}.$$

The (33) element in the matrix of AB , $(AB)_{33}$, is the inner product of the *third row* of A with the *third column* of B , i.e.

$$\begin{aligned} (AB)_{33} &= 2 \times (-3) + 3 \times (-5) + (-1) \times (-7) + (-2) \times 6 \\ &= -6 - 15 + 7 - 12 = -26. \end{aligned}$$

End Quiz